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THE USE OF DECISION THEORY IN METEOROLOGY

With an Application to Aviation Weather

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ABSTRACT

The concepts of decision theory are discussed, especially in the light of their application to meteorology. The use of the principles of decision-making under risk requires certain probability information to be available. The issuance of forecasts in probability terms has a firm basis in theory and has been shown to work well in practice. The best verification statistic of these forecasts is their usefulness to the user and this can be measured and compared with some standard if the utility matrix is known.

A multi-dimensional contingency table technique is used to estimate the conditional probability distribution of the 5-hr. projection of ceiling height at Washington National Airport. Three predictors are screened from 164 possible predictors according to the utility criterion. Developmental and test data results are presented.

1. INTRODUCTION

Meteorologists have ever been concerned with making better forecasts. There is little disagreement on what constitutes a good forecast; it is one that completely and accurately describes the weather element being forecast. However, since a series of these perfect forecasts is not attainable, it becomes necessary to have a measure of the "goodness" of a set of forecasts in order to know when one group of forecasts is really better than another. There is anything but agreement among meteorologists as to what measure should be used to make this judgment, and even as to how imperfect forecasts should be presented to the user. Decision theory provides a framework within which forecasts can be evaluated and at the same time suggests the form in which forecasts should be issued.

2. USE OF DECISION THEORY IN METEOROLOGY

CONCEPTS

Decision theory was introdued in 1939 by Wald [35] who published the first book on the subject in 1950 [36]; in it he formulated statistics as decision-making under uncertainty.

Consider the problem of an individual who needs to decide upon a course of action when several courses of action are available to him. He knows, or can estimate, what his utility (the numerical value of his action) is for each possible action and for each possible state of nature (future happening) relative to the problem. These utilities can be arranged in the form of a matrix and as such comprise a utility matrix. Conceptually, a utility matrix is shown in table 1; in this table U_{ij} is the utility for action A_j if state of nature Y_i occurs.

TABLE 1.—*A utility matrix. U_{ij} is the utility for action A_j if state of nature Y_i occurs*

State of Nature	Action					
	A_1	A_2	.	.	.	A_n
Y_1	U_{11}	U_{12}	.	.	.	U_{1n}
Y_2	U_{21}	U_{22}	.	.	.	U_{2n}
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
Y_m	U_{m1}	U_{m2}	.	.	.	U_{mn}

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TABLE 2.—*Conditional probability distribution of states of nature Y_i given the observations X_j*

State of Nature	Observations			
	X_1	X_2	. . .	X_p
Y_1	$P[Y_1\|X_1]$	$P[Y_1\|X_2]$	. . .	$P[Y_1\|X_p]$
Y_2	$P[Y_2\|X_1]$	$P[Y_2\|X_2]$	. . .	$P[Y_2\|X_p]$
.	.	.	. . .	.
.	.	.	. . .	.
.	.	.	. . .	.
Y_m	$P[Y_m\|X_1]$	$P[Y_m\|X_2]$	. . .	$P[Y_m\|X_p]$

TABLE 3.—*All possible strategies for the n possible actions and p observations*

Strategy	Observations					
	X_1	X_2	.	.	.	X_p
S_1	A_1	A_1	.	.	.	A_1
S_2	A_2	A_1	.	.	.	A_1
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
S_k	A_n	A_n	.	.	.	A_n

However, the individual is uncertain about the state of nature and must resort to past experience, an experiment, or some other source of information to obtain an estimate of the probability of each possible state of nature. These probabilities may be *a priori* probabilities $P[Y_i]$; or, if he is fortunate, he can accumulate data that will allow him to construct a table of conditional probabilities, called *a postericri* probabilities, of Y_i given the observations X_j, $P[Y_i|X_j]$. Such a table is shown in table 2.

Alternatively, the conditional probabilities $P[X_j|Y_i]$, along with the *a priori* probabilities $P[Y_i]$ will suffice (and indeed $P[Y_i|X_j]$ can be derived from $P[X_j|Y_i]$ and $P[Y_i]$ by Bayes Theorem [26]) and the decision problem is usually formulated in this manner. Although the conditional probabilities are shown here in tabular form, which indicates a discrete distribution, continuous distributions are not ruled out and may be known for some problems.

The individual now needs to formulate a strategy (a rule for decision making) which will indicate what action to take for each possible observation X_j. All possible strategies can be arranged as shown in table 3.

From the total of $k=n^p$ distinct strategies, the problem is to find the best one. Suppose that the probabilities $P[X_j|Y_i]$ are available and let $U(S_j,Y_i)$ represent the expected utility if strategy S_j is adopted and the state of nature Y_i occurs. Then

$$
\begin{aligned}
U(S_1,Y_i)&=U_{i1}P[X_1|Y_i]+U_{i1}P[X_2|Y_i]+\ldots+U_{i1}P[X_p|Y_i]\\
U(S_2,Y_i)&=U_{i2}P[X_1|Y_i]+U_{i1}P[X_2|Y_i]+\ldots+U_{i1}P[X_p|Y_i]\\
&\vdots\\
U(S_k,Y_i)&=U_{in}P[X_1|Y_i]+U_{in}P[X_2|Y_i]+\ldots+U_{in}P[X_p|Y_i]
\end{aligned}
\tag{1}
$$

for $i=1, 2, \ldots, m$. This gives a total of km expected utilities, one for each possible strategy and each possible state of nature.

If $U(S_1,Y_i)\geq U(S_2,Y_i)$ for all i and the inequality holds for at least one value of i, S_1 is said to dominate S_2. This means that no matter which state of nature occurs the strategy S_1 will yield on the average as high or higher utilities than S_2. In this case S_2 is called an inadmissible strategy; all strategies not dominated by one or more other strategies are admissible.

If the *a priori* probabilities of the states of nature are available, the best strategy (or one at least as good as all the rest) can be selected from all admissible ones by computing the expected value of the utility $U(S_j)$ for each of the strategies and choosing the one $U(S_t)$ which is at least as large as all the rest.

$$U(S_j)=\sum_{i=1}^{m} U(S_j,Y_i)P[Y_i] \tag{2}$$

The strategies S_j (of which S_t is one) which are used in computing the expected utilities $U(S_j)$ are called Bayes strategies and it is shown by Chernoff and Moses [8] that (1) every admissible strategy is a Bayes strategy for some set of *a priori* probabilities $\left(P[Y_i]\geq 0 \text{ and } \sum_{i=1}^{m} P[Y_i]=1\right)$, (2) not all Bayes strategies corresponding to the probabilities $P[Y_i]\geq 0$ and $\sum_{i=1}^{m} P[Y_i]=1$ may be admissible but if $P[Y_i]$ are limited to greater than zero then the corresponding Bayes strategies are admissible, and (3) a randomized Bayes strategy (a strategy that is a random mixture of two or more pure Bayes strategies) with the probability set $P[Y_i]$ may dominate a pure Bayes strategy that was admissible when only pure Bayes strategies were considered, but there is at least one pure Bayes strategy corresponding to that probability set that is not dominated by any randomized Bayes strategy. The result of these proofs is that only pure admissible Bayes strategies need be considered when it is desired to maximize the expected utility.

Basically, decision-making involving states of nature falls into three categories: (1) decision-making under certainty, which occurs when the state of nature is known with certainty, (2) decision-making under risk which occurs when the probability of occurrence of each of the states of nature is known, and (3) decision-making under uncertainty when the probabilities of the states of nature are not known.

Criteria other than that of Bayes exist for choosing the best strategy, but if the problem falls into the category of decision-making under risk and if the utility matrix contains the true utilities which reflect all pertinent aspects of the problem and not just the money involved, the Bayes solution is the only one that need be considered [21]. If the problem is one of decision-making under uncertainty, an unconditional expected utility cannot be defined [1].

In decision-making under risk it is possible to select the best action for each observation X_j separately. The selection can be done by computing

$$U(X_j, A_h)=\sum_{i=1}^{m} U_{ih}P[Y_i|X_j] \tag{3}$$

for each action A_h, $h=1, 2, \ldots, n$, and then selecting the action which maximizes $U(X_j, A_h)$, the expected utility when observation X_j occurs and action A_h is taken. If there are no observations X_j, then $P[Y_i|X_j]$ can be replaced by $P[Y_i]$ to obtain a constant course of action.

THE NEED

Whenever a weather forecast is made for a user it should be assumed that that user is going to make an operational decision based, at least in part, on the forecast. It should be the responsibility of the forecaster to impart as much information as possible concerning the weather element or elements in which the user is interested. If it were possible to predict a weather element perfectly, no question would arise as to how the information should be presented; a categorical forecast would contain all of the information.

Even if the atmosphere is considered as a deterministic system and the probability of a weather event is either zero or one, not all of the conditions which determine this future state are known. Under these imperfect conditions there is a conditional probability distribution of the weather event which contains all of the information concerning the event furnished by the known initial conditions. It has been shown by Schroeder [30], Sanders [28, 29], and Root [27] that forecasters can make rather good estimates of these conditional probabilities. It has also been shown by Brier [6], Thompson [31], and Dickey [11], to mention a few, that objective forecasting techniques are useful for this purpose.

There is increasing recognition among meteorologists of the desirability of presenting forecasts in probability terms. The inaccuracies of forecasts have long been recognized, as evidenced by the use of such terms as "scattered showers" and "occasional ceilings below 200 ft." However, these are vague terms and it is difficult even to persuade the forecasters to attach probabilities to them, let alone to persuade the users to interpret them in this light.

It is many times said, whenever a concurrent forecast-verification program is being conducted, that the forecasters are more concerned with beating the verification system than with making good forecasts. If this statement is true, the verification system is at fault and does not measure the "goodness" of the forecasts. How can the "goodness" of forecasts be measured? This is a question that must be answered by each user and the answer will reflect that user's particular utility matrix. Obviously, the user will want to make the best decision possible and decision theory provides a framework in which to work. At the same time, the verification statistic for the set of forecasts is suggested.

Although it is the user who must ultimately make the decision for his course of action, the meteorologist usually needs to concern himself with the decision problem for one or more reasons. First, the user is many times not well versed in the use of the information which the meteorologist can furnish him and needs advice along these lines. Second, the meteorologist wants to furnish a set of unbiased conditional probabilities to the user. The user may not care what observations went into the analysis; he is willing to accept the meteorologist's word that the (conditional) probabilities are correct. However, there are usually many observations available to the meteorologist and it is his problem to choose the ones to use in order that his conditional probabilities furnished the user will be as useful as possible. A knowledge of the utility matrix will help him decide which observations to use. Third, although there is only one true set of conditional probabilities for a given set of observations, these population values are not known and must be estimated from a data sample. This data sample can be analysed in many ways and not all analyses will yield the same estimate of conditional probabilities. A knowledge of the utility matrix will help the meteorologist to decide upon a method of analysis.

HISTORICAL DEVELOPMENT

Pioneering studies in the use of decision theory principles applied to meteorological problems are those of Bilham [3], Brier [5], Bijvoet and Bleeker [2], Thompson [31, 32], and Crossley [9]. Thompson and Brier [34] considered a 2×2 cost (or negative monetary utility) matrix which is comprised of the cost C of one level of protection and the loss L when no protection is accomplished for each of two possible weather outcomes adverse and good. When the conditional probability of adverse weather is greater than C/L or less than C/L, the action should be to protect or not protect respectively. They also devised the score, "saving over climatology," which is the amount of money that is saved, or lost, per dollar potential loss when a series of conditional probability forecasts is used over that saved when the climatological expectancies (*a priori* probabilities) are used. This score, therefore, provides a measure of the savings or usefulness of a series of forecasts and at the same time compares it with a standard, climatology.

In recent years, several other studies have been made in which the use of meteorological information is analyzed within the framework of decision theory. Borgman [4] analyzed an oil well drilling operation and observed that

"Accuracy [of forecasts] is desirable but is not sufficient to guarantee utility." Nelson and Winter [25] considered the problems of a truck dispatcher in "tarping" or not "tarping" the loaded fleet overnight, of a newspaper circulation manager in deciding whether to cover the papers for outside delivery, of the director of a motion-picture studio in scheduling outdoor and indoor scenes, and of a building contractor in scheduling workmen for pouring concrete. Kolb and Rapp [18] and Lave [19] treated the impact of weather information on the economics of the raisin industry and came to the conclusion that the use of improved weather information by a single user could result in increased profits; however, the latter author states that if the industry as a whole used the improved information, "The inelasticity of demand causes profit to fall . . ., at least in the short run." For rainfall forecasts made at San Francisco, Root [27] showed that with a C/L of 0.10, forecasts made in probability terms provided a higher saving than did climatology for both projections, 0–12 hr. and 36–48 hr., but that the categorical forecasts showed a higher saving than climatology for only the shorter projection. Demsetz [10] concluded in a study of tropical storm protection measures of the city of Miami and electrical service restoration by the Florida Power & Light Company, that improved tracking of tropical storms could be of substantial value but that the economic gains derivable from existing weather information are probably not being realized. The latter point of view has been generalized by Thompson [33] who showed that for each of three analyzed forecast problems the gain that can be realized by presentation of the forecasts in probability terms and the educated use of these forecasts is a substantial fraction of the gain that perfect forecasts would allow, except for values of C/L near the arbitrarily selected categorical decision level.

Gleeson [15] considered the multi-class predictand problem in which the upper and lower confidence limits of the relative frequencies of these classes are known. Gringorten [16, 17] addressed the problem of estimating the conditional probabilities in a manner that will best benefit a particular user and states, "In theory, at least, the issuing of one probability statement on a single day is not the most useful method to meet every operational requirement." He concluded that the purpose of the analysis of the data should be to minimize errors of estimate of operational gains rather than errors of estimate of conditional probabilities.

3. CONDITIONAL PROBABILITY ESTIMATION AND PREDICTOR SELECTION

Conditional probabilities can be estimated subjectively or objective techniques can be employed. Some of the techniques which use historical data and estimate conditional probabilities by some variation of the relative frequency concept are scatter diagrams [6], regression [24, 20, 23, 17], and discriminant analysis [22]. With the latter two of these techniques predictors can be selected objectively from a much larger set of possible predictors according to their ability to give good probability estimates; with scatter diagrams the selection of predictors is usually more subjective.

Multi-dimensional contingency tables [13] can be used for estimating conditional probabilities and with the use of high-speed computers predictor selection according to some desired criterion can also be made. Because of scarcity of data for some predictor category combinations, smoothing over neighboring cells of the contingency table is usually necessary.

4. AN APPLICATION TO THE 5-HR. PROJECTION OF CEILING HEIGHT

Conditional probablilities of five operationally significant classes of ceiling height (shown in table 4) at Washington National Airport have been estimated with the use of multi-dimensional contingency tables. Stepwise predictor selection from 164 possible predictors was made according to the utility criterion. Each possible predictor was used separately to determine $P[Y_i|X_j]$. Then the maximum of $U(X_j,A_h)$, $h=1, 2, \ldots, n$, was found for each sample point and this maximum summed over all sample points. The variable that yielded the highest total expected utility was selected as the first predictor. Then each possible predictor, excluding the first one chosen, was used with the first to again compute total utilities over the sample. The variable which together with the first produced the highest utility was chosen as the second predictor. This procedure was continued until a total of three predictors had been selected.

At each pth predictor selection a $p+1$ dimensional table was formed. The $(p+1)$th dimension corresponded to the predictand. Each predictor was in categorical form and for each sample point a count was entered in the cell of the table corresponding to the predictor and predictand categories. Then for each predictor category combination the conditional probability of each predictand category was defined by the relative frequency of that predictand category to the total observations for that particular predictor combination.

When more than one predictor was used, the scarcity of observations for some predictor category combinations was a problem and smoothing over surrounding cells became necessary. Smoothing rules, based partly on intermediate results, were made as the study progressed. In general, when the number of observations in a particular predictor category was less than k, observations in surrounding cells were included in the conditional probability estimates. The value of k used for predictor selection was 10. It was also necessary to specialize the

TABLE 4.—*The five classes of ceiling height used as a predictand*

Category	Ceiling Height (ft.)
1	0– 100
2	200– 400
3	500– 900
4	1000–2900
5	$\geq$3000

smoothing to each type of predictor. It was thought that the small gain in utility that could be expected from a fourth predictor did not justify the computing time necessary to smooth a five-dimensional table.

The complete list of 164 possible predictors is not included here. Briefly, the predictors were meteorological variables observed hourly at the surface of the earth at Washington National Airport and nine surrounding stations, Atlantic City, N.J., Norfolk, Va., Williamsport, Pa., Martinsburg, W. Va., Gordonsville, Va., Patuxent River, Md., Annapolis, Md., Roanoke, Va., and Pittsburgh, Pa., and the time of day and day of year of the observation. The elements included for one or more of the stations were ceiling height, visibility, west wind component, south wind component, temperature, dew point, relative humidity, sea-level pressure, amount of cloud in lowest layer, amount of cloud in the second layer, total cloud amount, opaque cloud amount, type of cloud in lowest layer, height of lowest cloud layer, height of second cloud layer, precipitation, fog, stability of air mass, wind speed, and wind direction. (A complete description of the 164 possible predictors is found in reference [14].)

The developmental sample included 4288 hourly observations taken during the 6-yr. period from January 1, 1949, through December 31, 1954. It was chosen in such a way that at least 5 hours elapsed between any two observations used and the observations were evenly distributed as to time of day.

The utility matrix shown in table 5 and used in this study was devised by R. A. Allen after consultation with forecasters at several aviation forecast centers. It is thought that this matrix may not be far different from that of an actual utility matrix of an airline and it was used by Enger, Reed, and MacMonegle [12, 13] for the purpose of evaluating ceiling height forecasts at seven terminals including Washington National Airport.

The three predictors selected are shown in table 6 together with the expected utility and P-Score [7] for each. A disadvantage of this probability estimation technique is that a great many degrees of freedom are used in the determination of $P[Y_i|X_j]$ unless extreme smoothing is done. However, if extreme smoothing is done, the resolution of which the data may be capable is lost. Although the predictor selection was done with k equal to 10, probabilities were also estimated with k equal to 25.

The tables which give the conditional probability distribution and the action which maximizes the expected utility for each predictor category combination are too large to be included here. The total utility for perfect forecasts is 957 for this sample. The entries in table 6 indicate that the third predictor, Martinsburg's ceiling height, furnishes little additional information over that furnished by the first two predictors, Washington's ceiling height and Martinsburg's wind direction.

TABLE 5.—*The utility matrix used to judge the usefulness of the forecasts*

Observed Category	Forecast Category				
	1	2	3	4	5
1	1.0	0.6	0.1	0.0	0.0
2	.7	.9	.4	.05	.0
3	.2	.5	.7	.2	.0
4	.0	.1	.3	.45	.1
5	.0	.0	.05	.1	.15

TABLE 6.—*The expected utilities and P-Scores for the 3 predictors selected according to the utility criterion*

Order of Predictor Selection	Predictor	Expected Utility		P-Score	
		$k=10$	$k=25$	$k=10$	$k=25$
1	Washington, ceiling	759	759	0.218	0.218
2	Martinsburg, wind direction.	784	776	.205	.211
3	Martinsburg, ceiling	793	776	.197	.205

5. INDEPENDENT DATA TESTING

An 8632-case sample consisting of all usable hourly observations taken during the period October 1, 1960, through September 30, 1961, was used for test purposes. From the conditional probability tables, P-Scores were computed and from the action tables actual utilities were found. The results are shown in table 7.

The ceiling heights were in general higher during the test data period than during the developmental data period and since high ceilings are usually easier to forecast than low ceilings, the P-Scores are better for the test data than for the developmental data. The utility for perfect forecasts is 1748.

The test results indicate that the minimum number of observations required for the probability determination should be greater than 10. Although the P-Scores show that the second and third predictors furnish information, the utility added by these two predictors is small.

6. SUMMARY AND CONCLUSIONS

The use of the principles of decision-making under risk requires certain probability information to be available. The issuance of forecasts in probability terms has a firm basis in theory and has been shown to work well in practice. The best verification statistic of these forecasts is their usefulness to the user. This can be measured and compared with some desired standard if the utility matrix is known.

The results of the use of multi-dimensional contingency tables for conditional probability estimation were somewhat disappointing in that little more utility was afforded

TABLE 7.—*Actual utilities and P-Scores for test sample*

Predictor(s)	Actual Utility		P-Score	
	$k=10$	$k=25$	$k=10$	$k=25$
Washington, ceiling	1450	1450	0.166	0.166
Washington, ceiling; Martinsburg, wind direction	1444	1457	.166	.161
Washington, ceiling; Martinsburg, wind direction; Martinsburg, ceiling	1450	1457	.163	.160

on test data by three predictors selected specifically for that purpose than by only the first predictor. Also, the contingency table method becomes very cumbersome for more than three predictors and even if very large samples were available for development, large amounts of computer time would be needed for probability determination. Other studies conducted by the author, with the same developmental and test data samples used in this study, indicate that some suitable parametric technique, such as multiple discriminant analysis, has more to offer for a prediction problem of this kind than does this non-parametric contingency table method.

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